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TECHNICAL NOTES

Linear stability study of waviness effect on film condensation with nonuniform surface tension

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1. INTRODUCTION

The waviness effect on film condensation has been analytically investigated utilizing linear stability theory by many researchers [1, 2]. In their studies, the temperature of the film surface is taken to be constant. However, in general, the surface temperature varies in time and space as the film surface fluctuates. The temperature fluctuation leads to the surface tension variation which has profound effects on the film stability. Actually, surface tension also plays a role in the generation of liquid motion when the liquid-vapor interface has finite curvature or when surface tension varies over the interface due to gradients in temperature. Lin [3] included the temperature fluctuation in the disturbance equations and found that the surface tension stabilizes condensate films but destabilizes liquid film on a heated wall. However, from the criticisms stated by Spindler *et al.* [4], the mass and momentum jump conditions due to interfacial phase change were misrepresented in Lin's studies. Ünsal and Thomas [5], for a condensation film flow only, accurately took into account the phase change using a modified Yih's analysis, but still made some unjustified simplifications and neglected temperature variation effects. Spindler [6] studied the influence of the interfacial phase change on the stability of liquid film over an inclined plane. The thickness variation caused by the phase change was also taken into account.

In the present study, a linear stability analysis is employed to assess the waviness effect on film condensation over a vertical plane. The authors accounted for the effects of the surface tension caused by the surface temperature fluctuation through wavy motion. The analytical approach is restricted to two-dimensional periodical waves at low film Reynolds numbers. The film Reynolds numbers of the laminar-wavy flow regime, in general, are limited from 20 to 450. The surface tension is assumed to decrease linearly with the temperature and expressed as $\sigma = \sigma_0 - \lambda_c(T - T_S)$. The resulting stability problem is analyzed by an approximate method to yield asymptotic solutions.

2. ANALYSIS

The present study relates to laminar liquid film flow down a cooled vertical plate and assumes that the condensation process is slow. It is assumed that the wave presents on the film surface is a long wave in an equilibrium condition and locally has a constant amplitude. The governing equations of mass, momentum, and energy for the laminar film of two-dimensional periodic waves with constant properties are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + (1-\gamma)g, \tag{1b}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \tag{1c}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{1d}$$

with the associated boundary conditions:

$$\text{at } y = 0: \quad T = T_w, \quad u = v = 0, \tag{2a,b,c}$$

$$\text{at } y = \delta: \quad v - u\frac{\partial \delta}{\partial x}(1-\gamma) - \frac{\partial \delta}{\partial t}(1-\gamma) = \gamma v_g, \tag{2d}$$

$$P - P_g = \rho\frac{1-\gamma}{\gamma}\left(v - u\frac{\partial \delta}{\partial x} - \frac{\partial \delta}{\partial t}\right)^2 + 2\mu\frac{\partial v}{\partial y} - \sigma\frac{\partial^2 \delta}{\partial x^2}, \tag{2e}$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + 4\frac{\partial \delta}{\partial x}\frac{\partial v}{\partial y} = \frac{1}{\mu}\frac{d\sigma}{dT}\frac{\partial T}{\partial x}, \tag{2f}$$

$$k\left(\frac{\partial T}{\partial y} - \frac{\partial T}{\partial x}\frac{\partial \delta}{\partial x}\right) - \rho h_{fg}\left(\frac{\partial \delta}{\partial t} + u\frac{\partial \delta}{\partial x} - v\right) = 0. \tag{2g}$$

The similar dimensionless variables and asymptotic expansions for long waves in ref. [7] are introduced. Thus, the asymptotic solutions of the zeroth and first-order problems can be readily solved and the derivation details are omitted here. The expression for the condensation rate is given by

$$\xi\left[\frac{1}{d} - \alpha PrRe\left(\frac{1}{3}d_t + \frac{33}{40}d^2 d_s\right)\right]\left(1 + \alpha^2 d_s^2\right)$$

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NOMENCLATURE

c	complex wave celerity, $c_r + ic_i$
Ca	capillary number, $\lambda_c(T_w - T_s)/\sigma_0$
d	dimensionless film thickness, δ/δ_m
Nd	dimensionless condensation parameter, ξ^2/γ
N_σ	surface tension parameter, $[2\sigma^3/(\rho^3 v^4 g(1-\gamma))]^{1/3}$
Re_c	critical Reynolds number
s	dimensionless x-coordinate, $\alpha x/\delta_m$
T_s	vapor saturation temperature
T_w	wall temperature
ΔT	temperature difference, $T_s - T_w$
u_m	local base flow interfacial liquid x-component velocity, $g(1-\gamma)\delta_m^2/2v$
We	Weber number, $\sigma_0/(\rho\delta_m u_m^2)$
x_c	critical distance.

Greek symbols

α	dimensionless wave number
δ	film thickness
δ_m	local base flow film thickness, $[4kv\Delta Tx/((1-\gamma)\rho g h_{fg})]^{1/4}$
ε	infinitesimal disturbance
γ	ρ_g/ρ
λ_c	$-\mathrm{d}\sigma/\mathrm{d}T$
τ	dimensionless time
σ	surface tension, $\sigma_0 - \lambda_c(T - T_s)$
σ_0	surface tension at saturation temperature T_s
ξ	acceleration parameter, $k\Delta T/(\mu h_{fg})$
ζ	heat capacity parameter, $c_p\Delta T/h_{fg}$.

$$
\begin{aligned}
= \alpha \mathrm{Re}\Bigg\{ & \mathrm{d}_\tau + 3\mathrm{d}^2\mathrm{d}_s + \alpha\mathrm{Re}\left[\frac{2}{3}\frac{1-\gamma}{\gamma}\left(\frac{\xi}{\mathrm{Re}}\right)^2 \mathrm{d}_{ss}\right. \\
& + \alpha^2 We d^2 d_s d_{sss} + \frac{1}{3}\alpha^2 We d^3 d_{ssss} \\
& - \frac{1}{2} WeCa(d_s^2 + dd_{ss}) \\
& \left.\left. - \frac{5}{2}\mathrm{d}^3\mathrm{d}_s\mathrm{d}_\tau - \frac{5}{8}\mathrm{d}^4\mathrm{d}_{s\tau} - \frac{81}{20}\mathrm{d}^5\mathrm{d}_s^2 - \frac{27}{40}\mathrm{d}^6\mathrm{d}_{ss}\right]\right\} \qquad (3)
\end{aligned}
$$

The stability of liquid film can be studied by linearizing eqn (3). Consider an infinitesimal disturbance superimposed on the base flow. Let $d = 1+\varepsilon$, where ε is an arbitrary small variable. After substituting $d = 1+\varepsilon$ into eqn (3), and letting $\varepsilon = \exp[i(s - c\tau)]$ as the normal mode, the following expressions for the dimensionless wave velocity and the dimensionless wave amplification rate are obtained.

$$
\begin{aligned}
c_r = \frac{5}{8}\Bigg[& \frac{24}{5} + \xi\left(1 + \frac{73}{25}Pr\right) + \xi^2\left(\frac{11}{25}Pr^2 - \frac{2}{3}\frac{1-\gamma}{\gamma}\alpha^2\right) \\
& - \alpha^2 Re^2\left(\frac{1}{3}\alpha^2 We + \frac{1}{2}WeCa + \frac{27}{40}\right)\Bigg] \Bigg/ \\
& \left[\left(1+\frac{\zeta}{3}\right)^2 + \left(\frac{5}{8}\alpha Re\right)^2\right] \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
\alpha c_i = \Bigg[& -\left(1+\frac{\zeta}{3}\right)\frac{\zeta}{Pe} + \alpha^2 Re\left(\frac{6}{5} + \frac{93}{320}\zeta\right) \\
& + \frac{2}{3}\alpha^2(1-\gamma)\frac{Nd}{Re}\left(1+\frac{\zeta}{3}\right) - \frac{1}{3}\alpha^4 WeRe\left(1+\frac{\zeta}{3}\right) \\
& - \frac{1}{2}WeCa\alpha^2 Re\left(1+\frac{\zeta}{3}\right)\Bigg] \Bigg/ \left[\left(1+\frac{\zeta}{3}\right)^2 + \left(\frac{5}{8}\alpha Re\right)^2\right]. \qquad (5)
\end{aligned}
$$

The neutral stability curve can be obtained by letting $c_i = 0$, resulting in relation between the dimensionless wave number α and the Reynolds number along the curve. To solve the resulting equation for $\alpha c_i = 0$ with small ζ assumption, eqn (5) gives the critical Reynolds number expressed in the following form.

$$
Re_c^2\left(\frac{6}{5} - \frac{1}{2}WeCa\right) + \frac{2}{3}\frac{1-\gamma}{\gamma}\xi^2 = \left(\frac{4}{3}\xi N_\sigma\right)^{1/2} Re_c^{1/6}. \qquad (6)
$$

For most cases, the surface tension on the right-hand side of eqn (6) is one order of magnitude larger than the second term on the left-hand side. Therefore, eqn (6) can be further simplified with negligible loss of accuracy to

$$
Re_c = \left[\frac{\left(\frac{4}{3}\xi N_\sigma\right)^{1/2}}{\frac{6}{5} - \frac{1}{2}WeCa}\right]^{6/11}. \qquad (7)
$$

When surface tension is uniform (i.e. $Ca = 0$), eqn (7) becomes

$$
Re_c = \left[\frac{25}{27}\xi N_\sigma\right]^{3/11}. \qquad (8)
$$

Observing that

$$
Re = \frac{u_m\delta_m}{v} = \frac{g(1-\gamma)}{2v^2}\left[\frac{4v^2\xi x}{(1-\gamma)g}\right]^{3/4} \qquad (9)
$$

the critical distance can be solved from eqns (7) and (9), and gives

$$
x_c = \frac{1}{4}\left[\left(\frac{3\rho}{64\sigma_0}\right)^4 v^{-2}g^5(1-\gamma)^5 \times\left(\frac{6}{5} - \frac{1}{2}WeCa\right)^8 \xi^7\right]^{-1/11}. \qquad (10)
$$

Again, when the surface tension is uniform, the critical distance becomes

$$x_c = \frac{1}{4}\left[\left(\frac{400}{27}\sigma_0\right)^4 \left(\frac{c_p}{k\zeta}\right)^7 \frac{\rho^3 \nu^9}{g^5(1-\gamma)^5}\right]^{1/11}. \qquad (11)$$

3. RESULTS AND DISCUSSIONS

In the present study, it is obvious from eqns (5) and (7) that the increase in $WeCa$ (characterized as the surface tension effect) results in a decrease in αc_i and an increase in Re_c. It means that the temperature fluctuation leads to the surface tension variation has a stabilizing effect on the film stability. This result agrees with Lin's study [3] who also presented the stabilizing effect by the surface tension variation. The comparison of the neutral stability curves between Ünsal and Thomas and the present study is illustrated in Fig. 1. For easy comparison, the same saturated steam temperature at 373 K with $\Delta T = 47$ K in ref. [5] is selected. It is seen that the calculated critical Reynolds number in the present study ($Re_c = 5.0$) has little difference with that in Ünsal and Thomas' result ($Re_c = 7.9$) but achieves good agreement in their shapes.

As discussed previously, the surface tension variation due to temperature fluctuation has a stabilizing effect on the film stability. It can also be shown evidently in Fig. 2 that the effect of nonuniform surface tension is significant and the calculated critical Reynolds number Re_c ($=51.9$) is approximately ten times larger than the value of uniform surface tension under the same vapor-liquid conditions. The present work also extends the studies of Ünsal and Thomas to include the effects of the surface tension variation caused by the temperature fluctuation [8] correctly. Following their approach, the calculated critical Reynolds number and the critical distance can be summarized in the Appendix. It is seen from Fig. 2 that the surface tension variation effects evaluated by the extended Ünsal and Thomas' approach also have the same significant effect. The calculated Re_c ($=84.2$) is also approximately 10 times larger than the original Ünsal and Thomas' result. Besides, Lin's result [3] is also shown in Fig. 2 for comparison. The calculated Re_c ($=37.0$) of Lin is lower than the value of the present study, however, the order is the same and the shape of the neutral stability curve is similar. The discrepancy of Lin's studies is caused by the misrepresentation of the interfacial mass and momentum conditions.

Figures 3 and 4 depict the neutral stability curve of various temperature gradients for uniform and nonuniform surface tension, respectively. In these figures, the vapor temperatures are fixed at 373 K, yet a wide range of ΔT is considered. It is seen from Fig. 3 that the calculated critical Reynolds number changes only a little difference due to various temperature gradients ΔT from 10 to 60°C. This implies that the variation in temperature gradients affects the film stability insignificantly when surface tension is uniform. While for the case of nonuniform surface tension in Fig. 4, the differences in the neutral stability curves are distinct. The critical Reynolds number increases with increasing temperature gradient. It is obvious since higher values of ΔT mean larger condensation flow rate (ξ) thus signify greater surface tension. Consequently, the results indicate that the higher the condensation rate, the more stable is the film.

Some numerical calculations of Re_c and x_c at $T_s = 373$ K with various temperature gradients for both uniform and nonuniform surface tension are summarized in Table 1. The results of the left part (i.e. eqns (A1)–(A4) in Table 1 are calculated from the extended Ünsal and Thomas' results, whereas the right part (i.e. eqns (7)–(8) and (10)–(11)) shows the present study results. It is true that the flow is stable for disturbance generated at an abscissa lower than a critical

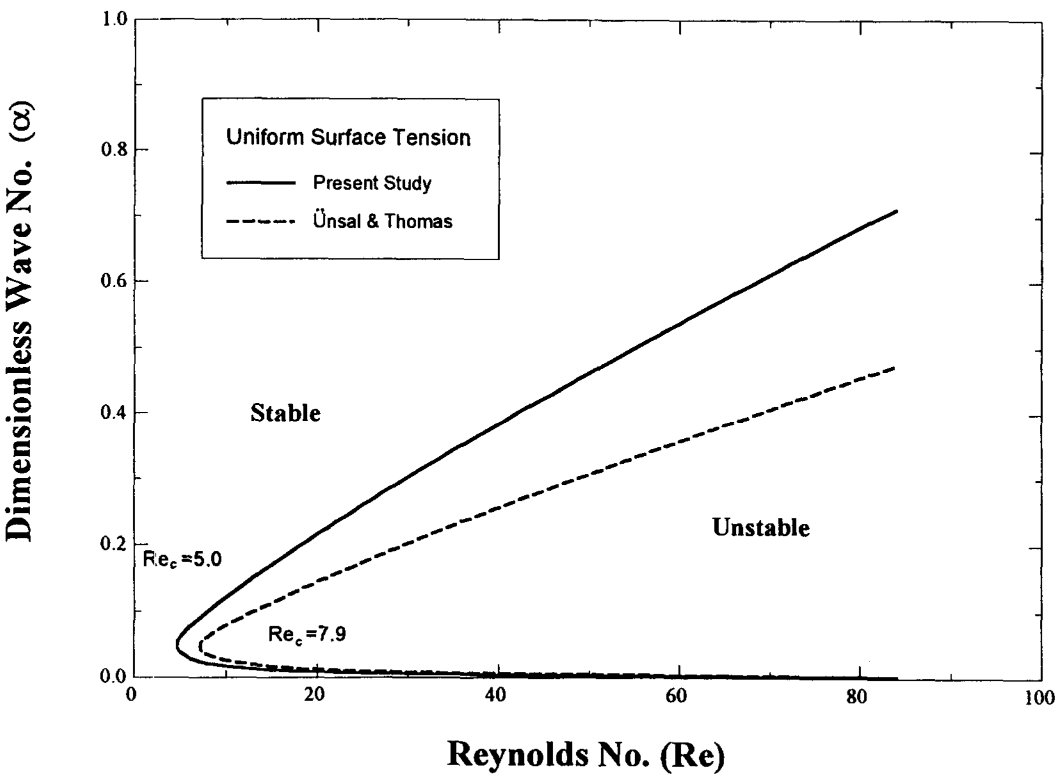

Fig. 1. Neutral stability curves for uniform surface tension at saturated temperature $T_S = 373$ K with $\Delta T = 47$ K.

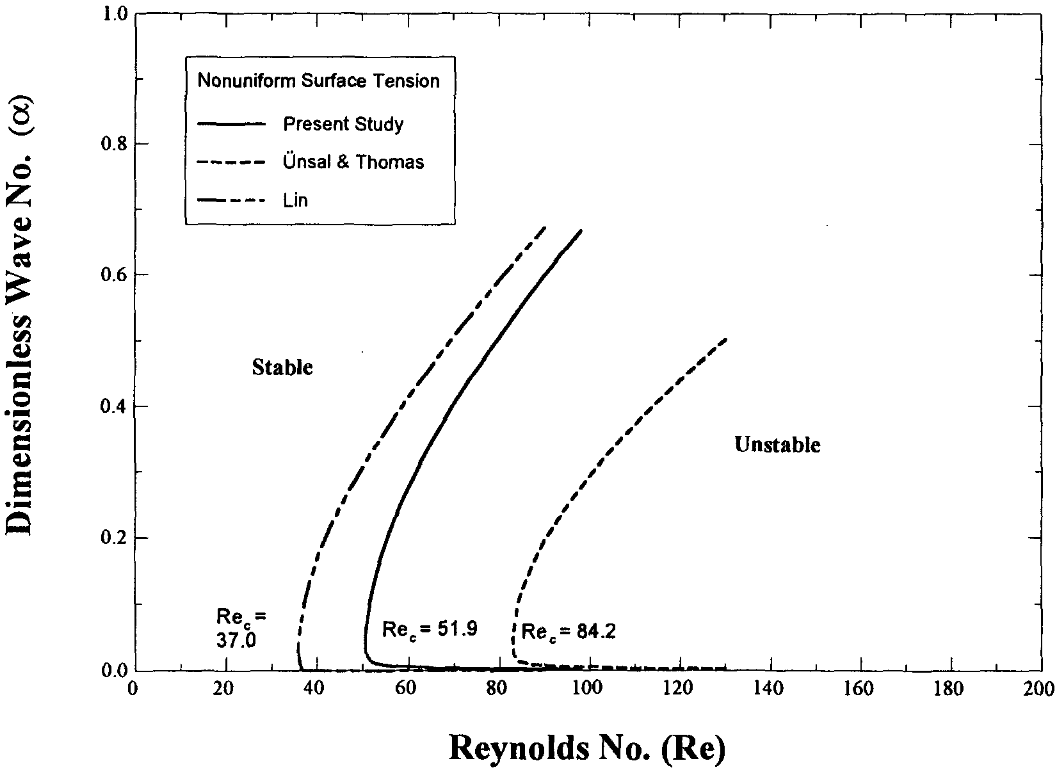

Fig. 2. Neutral stability curves for nonuniform surface tension at saturated temperature $T_S = 373$ K with $\Delta T = 47$ K.

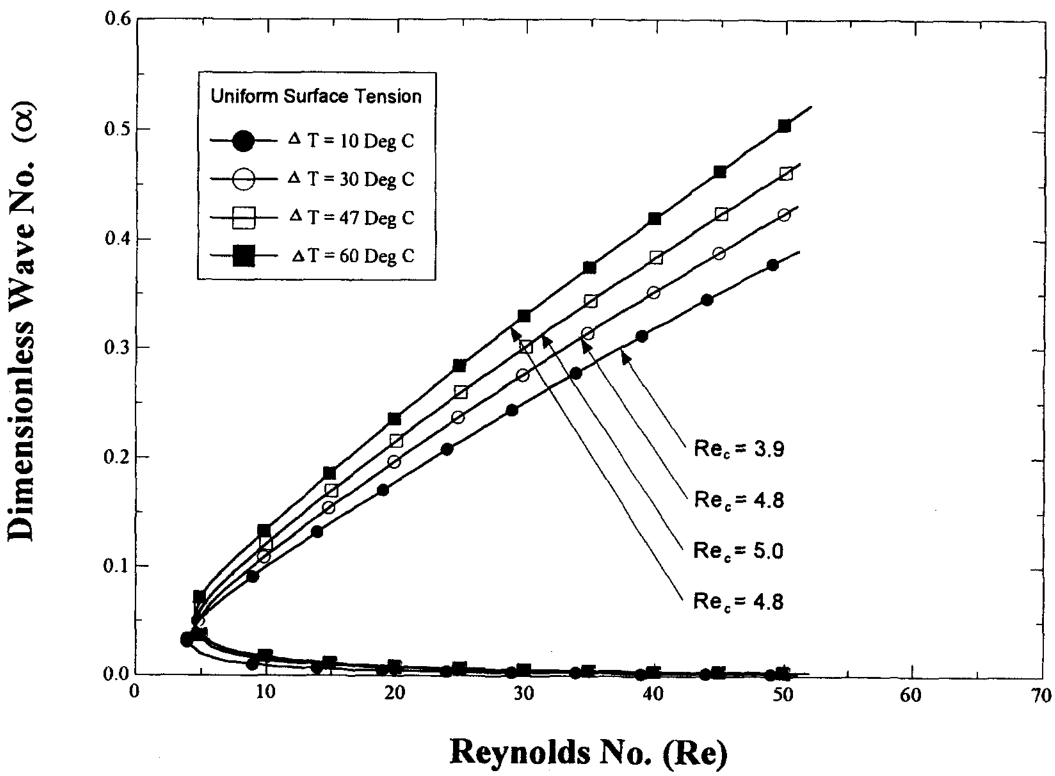

Fig. 3. Neutral stability curves for uniform surface tension at $T_S = 373$ K and various temperature gradients.

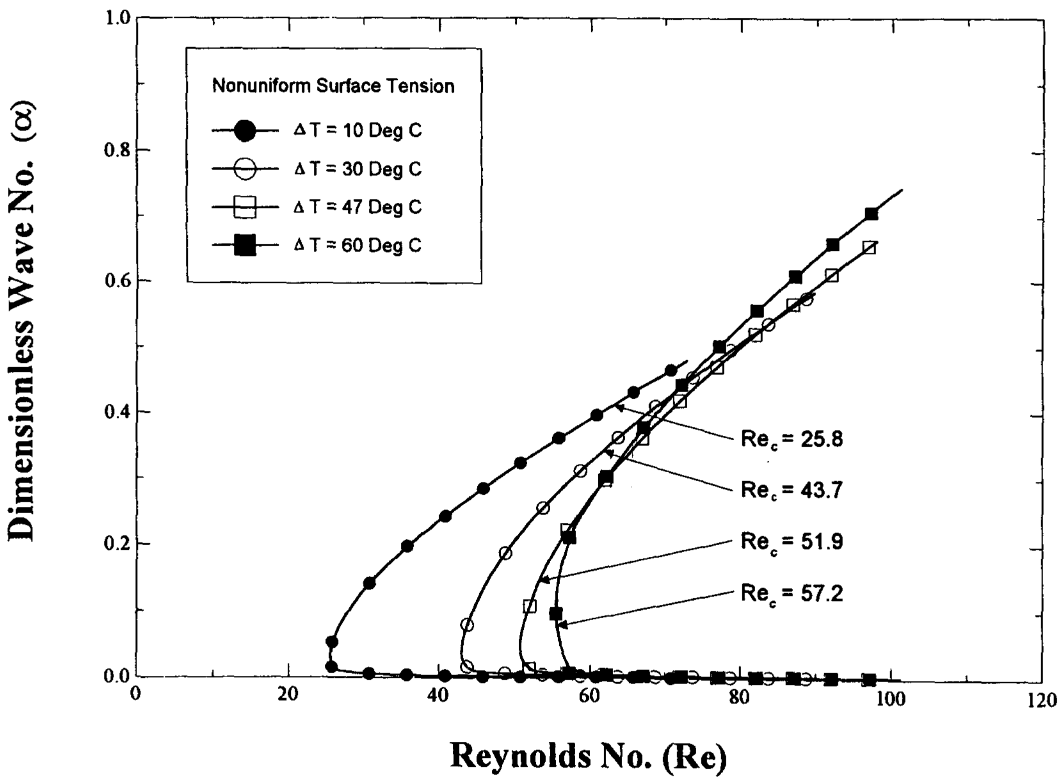

Fig. 4. Neutral stability curves for nonuniform surface tension at $T_S = 373$ K and various temperature gradients.

distance x_c for any wave number. From the results shown in Table 1, increasing Re_c does not ascertain the increase of the critical distance in both uniform and nonuniform surface tension cases. It exists because of the functional dependence of Re on ξ and We as shown by eqns (7) and (9). Equation (7) shows that increasing ξ increases Re_c which implies a stabilizing effect. However, eqn (10) shows that the critical distance varies inversely with ξ. Consequently, increasing ξ decreases the critical distance x_c and therefore has a destabilizing effect. From eqns (7) and (10), it is seen that increasing $1/2\,WeCa$ increases both Re_c and x_c which has a stabilizing effect.

4. CONCLUSION

In the present study, a linear stability analysis is employed to assess the waviness effect on film condensation over a vertical plane. The influences of the surface tension variation caused by the surface temperature fluctuation were correctly considered. The asymptotic solutions for the dimensionless wave amplification rate, the dimensionless wave velocity, the neutral stability curve, and the critical Reynolds number are obtained in this paper.

For nonuniform surface tension, the results show that the surface tension variation stabilizes condensate films and has a significant effect on the film stability. The critical Reynolds number prediction is about 10 times larger than the value of the uniform surface tension both in the present study and the extended results from Ünsal and Thomas. Besides, it is found that the variation in temperature gradients has little effect on the film stability for uniform surface tension. This is opposed to nonuniform surface tension for which the differences are distinct. The Re_c increases with increasing ΔT which indicates the higher the condensation rate, the more stable is the film.

Finally, the results show that increasing temperature gradi-

Table 1. Numerical calculations of Re_c and x_c at $T_s = 373$ K with various temperature gradients for both uniform and nonuniform surface tension

ΔT	Uniform surface tension				Nonuniform surface tension			
	Re_c		x_c [cm]		Re_c		x_c [cm]	
[°C]	Eqn (A2)*	Eqn (8)	Eqn (A4)*	Eqn (11)	Eqn (A1)*	Eqn (7)	Eqn (A3)*	Eqn (10)
10	6.11	3.92	1.55	0.86	41.28	25.78	19.87	10.40
30	7.48	4.79	0.88	0.49	71.43	43.68	17.35	9.08
47	7.90	5.05	0.73	0.40	84.23	51.94	17.31	9.05
60	7.58	4.84	0.71	0.39	88.46	57.18	18.10	9.46

* Equations (A1)–(A4) are shown in the Appendix.

ent decreases the critical distance but increases the critical Reynolds number. It is also seen that increasing surface tension increases both Re_c and x_c and has a stabilizing effect.

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APPENDIX

The authors extend Ünsal and Thomas' studies [5] and correctly consider the surface tension variation caused by the interfacial temperature fluctuation through wave motion. The calculated Re_c is derived as

$$Re_c = \left[\frac{(\frac{4}{3}\xi N_\sigma)^{1/2}}{\frac{8}{15}-\frac{1}{2}WeCa}\right]^{6/11}. \quad (A1)$$

When the surface tension is uniform (i.e. $Ca = 0$), the calculated Re_c shown in the following equation is the same as the result obtained by Ünsal and Thomas.

$$Re_c = \left[\frac{75}{16}\xi N_\sigma\right]^{3/11}. \quad (A2)$$

The critical distance is derived as

$$x_c = \frac{1}{4}\left[\left(\frac{3\rho}{64\sigma_0}\right)^4 v^{-2} g^5 (1-\gamma)^5 \left(\frac{8}{15}-\frac{1}{2}WeCa\right)^8 \xi^7\right]^{-1/11}. \quad (A3)$$

Similarly, for $Ca = 0$, eqn (A3) becomes

$$x_c = \frac{1}{4}\left[(75\sigma_0)^4 \left(\frac{c_p}{k\zeta}\right)^7 \frac{\rho^3 v^9}{g^5(1-\gamma)^5}\right)\Bigg]^{1/11}. \quad (A4)$$

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Correction functions for a wide range of measured substrate temperature histories

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INTRODUCTION

As highlighted by a number of recent studies [1–2], the design of high temperature components, ceramics in particular, is a difficult process because of the often conflicting thermal and mechanical criteria that define their service. These "conflicts" will no-doubt increase as machines and materials are pushed to their strength and temperature limits in the search for improved efficiency and cost effectiveness. Because of the increasingly severe conditions that components are expected to endure, proactive design methodologies using predictive thermal and stress models are necessary to expose and explore all threats to component safety during typical service conditions. Yet, any realistic modeling of these potentially